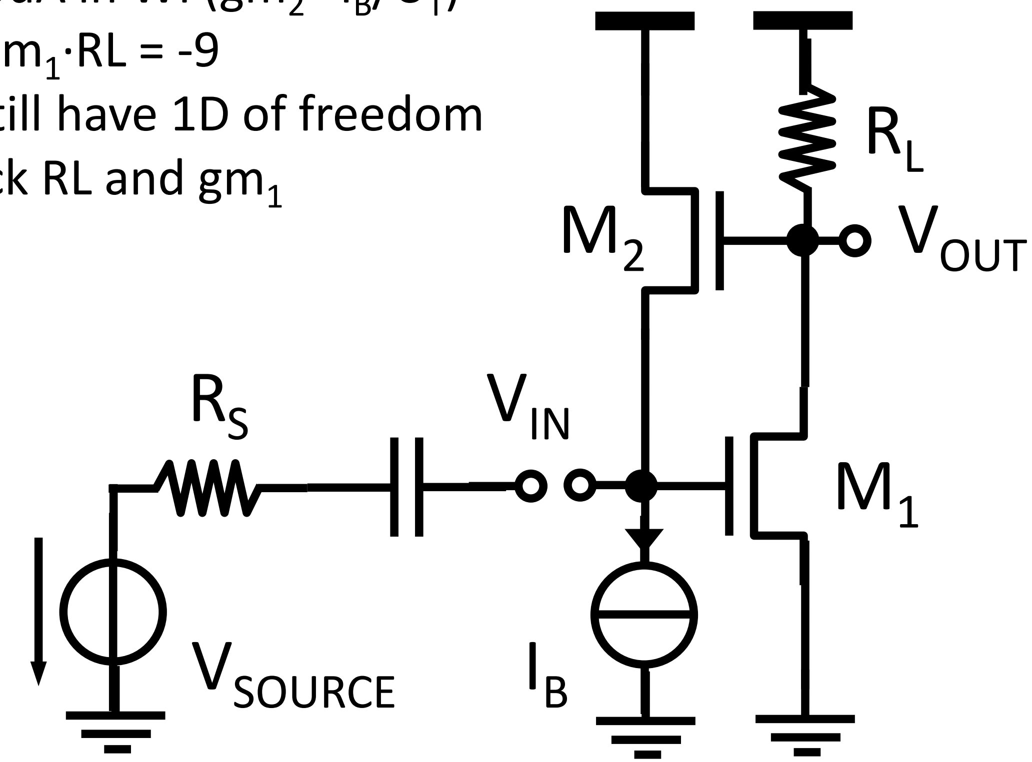
LNA design summary

15.11.23

ZIN & V gain calculation

Under input match condition:
Setting the voltage gain to e.g. -9 (19dB)
Imposes $gm_2 \cdot Rs = 0.1$ -> $gm_2 = 1/500\Omega = 2mS$
IB current could be sized vs IC
IB=50uA in WI ($gm_2 = I_B/U_T$)
$A_V = -gm_1 \cdot RL = -9$
We still have 1D of freedom
to pick RL and gm_1



KIRCHHOFF equations

$$\begin{pmatrix} I_{IN} \\ 0 \end{pmatrix} = \begin{pmatrix} g_{ms2} & -g_{m2} \\ g_{m1} & 1/R_L \end{pmatrix} \cdot \begin{pmatrix} V_{IN} \\ V_{OUT} \end{pmatrix}$$

$$A_V = \frac{V_{OUT}}{V_{IN}} = -g_{m1} \cdot R_L$$

$$Z_{IN} = \frac{V_{IN}}{I_{IN}} = \frac{1}{g_{ms2} + g_{m2} \cdot g_{m1} \cdot R_L}$$

If gms2=gm2

$$\frac{1}{g_{m2} \cdot (1 + g_{m1} \cdot R_L)} = R_S$$

Rewriting the input match condition

$$\frac{1}{(1 + g_{m1} \cdot R_L)} = g_{m2} \cdot R_S < 1$$

$$\frac{1}{(1 - A_V)} = g_{m2} \cdot R_S$$

If A_V=-9
$gm_2 \cdot Rs = 0.1$

$$\frac{V_{OUT}}{V_{SOURCE}} = \frac{V_{OUT}}{V_{IN}} \cdot \frac{\frac{V_{SOURCE}}{2 \cdot R_S} \cdot Z_{IN}}{V_{SOURCE}} = \frac{-g_{m1} \cdot R_L \cdot R_S}{2 \cdot R_S} = \frac{-g_{m1} \cdot R_L}{2} = \frac{A_V}{2}$$

Zout calculation with and without Rs

Without RS (conceptual, not real case) we get:
The injected output current is split between 1/RL and gm1 via the voltage follower formed by M2

$$\begin{pmatrix} 0 \\ I_{OUT} \end{pmatrix} = \begin{pmatrix} g_{ms2} & -g_{m2} \\ g_{m1} & {}^{1}/_{R_L} \end{pmatrix} \cdot \begin{pmatrix} V_{IN} \\ V_{OUT} \end{pmatrix} \quad Z_O = \frac{V_{OUT}}{I_{OUT}} = \frac{1}{g_{m1} + {}^{1}/_{R_L}}$$

If gms2=gm2

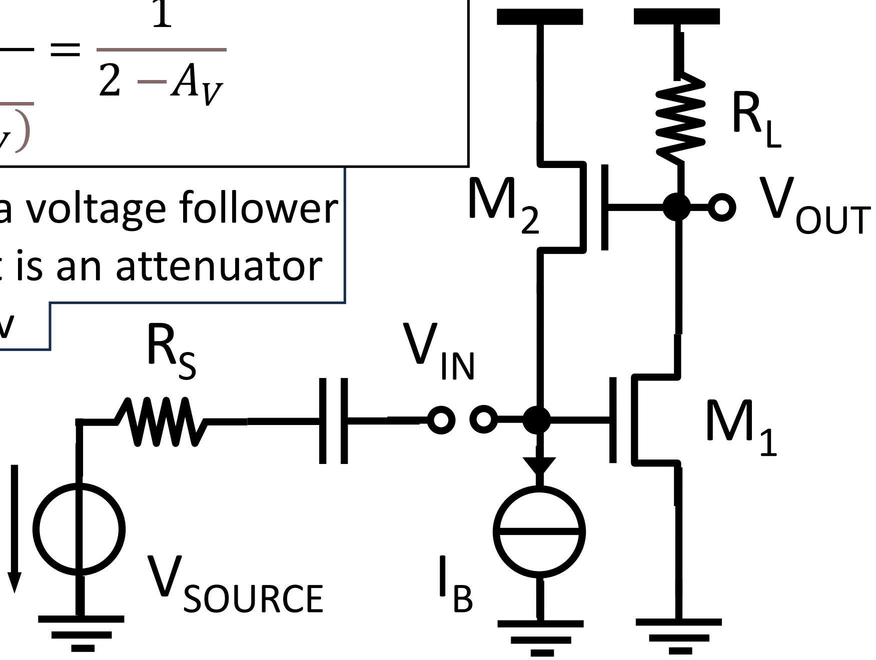
With R_S, the reverse voltage gain is

$$\frac{V_{IN}}{V_{OUT}} = \frac{g_{m2}}{{}^{1}/_{R_S} + g_{ms2}} = \frac{g_{m2} \cdot R_S}{1 + g_{m2} \cdot R_S}$$

If gms2=gm2

$$= \frac{\frac{1}{(1 - A_V)}}{1 + \frac{1}{(1 - A_V)}} = \frac{1}{2 - A_V}$$

If Rs >>, M2 is a voltage follower
With Rs=50Ω it is an attenuator by a factor 2-Av



With RS

$$\begin{pmatrix} 0 \\ I_{OUT} \end{pmatrix} = \begin{pmatrix} {}^{1}/_{R_S} + g_{ms2} & -g_{m2} \\ g_{m1} & {}^{1}/_{R_L} \end{pmatrix} \cdot \begin{pmatrix} V_{IN} \\ V_{OUT} \end{pmatrix}$$

$$Y_{OUT} = \frac{I_{OUT}}{V_{OUT}} = g_{m1} \cdot \frac{g_{m2}}{{}^{1}/_{R_S} + g_{ms2}} + {}^{1}/_{R_L}$$

If gms2=gm2

$$Z_{OUT} = \frac{1}{\frac{R_S \cdot g_{m2} \cdot g_{m1}}{1 + R_S \cdot g_{m2}} + {}^{1}/_{R_L}}$$

Now using Zin=Rs, we may remove gm1 or gm2

$$g_{m1} = \frac{1}{R_S \cdot R_L} \cdot \frac{1}{g_{m2}} - {}^{1}/_{R_L} \quad \frac{1}{R_S \cdot (1 + g_{m1} \cdot R_L)} = g_{m2}$$

If A_V=-9, gm_2·Rs = 0.1, Z_O~R_L/2 (2-Av)/(1-Av)=11/10=1.1

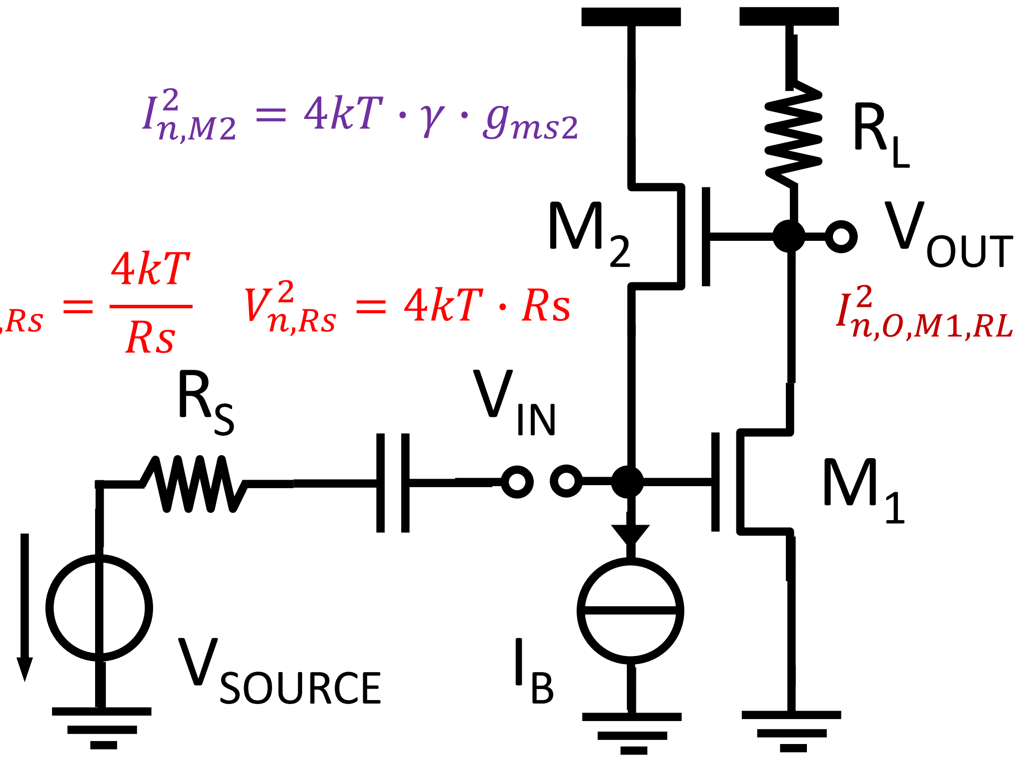
$$Z_{OUT} = \frac{R_L \cdot (1 + R_S \cdot g_{m2})}{2} \quad Z_{OUT} = \frac{R_L}{2} \cdot \frac{2 + g_{m1} \cdot R_L}{1 + g_{m1} \cdot R_L}$$

Equivalent noise calculation

$I^2_{n,M2} = 4kT \cdot \gamma \cdot g_{ms2}$

$I^2_{n,Rs} = \frac{4kT}{Rs}$ $\quad V^2_{n,Rs} = 4kT \cdot Rs$

$I^2_{n,O,M1,RL} = 4kT \cdot \left(\frac{1}{R_L} + \gamma \cdot g_{ms1}\right)$



We neglect 1/f noise from MOS, superposition applies
Use $V_n^2 = Z^2 \cdot I_n^2$ with Z_{OUT} and $Z_{IN}/2$ computed previously
Noise from IB current source to be added to that of M2!

$$V^2_{n,O,,M1,RL} = kT \cdot \left(\frac{1}{R_L} + \gamma \cdot g_{ms1}\right) \cdot \underbrace{R_L^2 \cdot (1 + R_S \cdot g_{m2})^2}_{/4 \; = \; Z_{OUT}^2}$$

$V_n^2 = ZOUT^2 \cdot In^2$

$$V^2_{n,in,M2} = kT \cdot \gamma \cdot g_{ms2} \cdot R_S^2$$

$$V^2_{n,O,M2} = kT \cdot \gamma \cdot g_{ms2} \cdot R_S^2 \cdot \overbrace{g_{m1}^2 \cdot R_L^2}^{A_V^2}$$

$V_n^2 = 1/4 \cdot ZIN_2 . In^2$ $\qquad V_{n,O}^2 = Vn_{,IN}^2 \cdot A_V^2$

Indeed we see Rs and Zin in parallel hence Rs/2
Multiply then by A_V^2 to get output noise @ V_{OUT}

Equivalent noise calculation

$$F = \frac{V_{n,O,RS}^2 + V_{n,O,M2}^2 + V_{n,O,M1,RL}^2}{V_{n,O,RS}^2}$$

$$= 1 + \frac{kT \cdot \gamma \cdot g_{ms2} \cdot R_S^2 \cdot g_{m1}^2 \cdot R_L^2 + kT \cdot \left(\frac{1}{R_L} + \gamma \cdot g_{ms1}\right) \cdot R_L^2 \cdot (1 + R_S \cdot g_{m2})^2}{kT \cdot R_S \cdot g_{m1}^2 \cdot R_L^2}$$

$$F = 1 + \gamma \cdot g_{ms2} \cdot R_S + \left(\frac{1}{R_L} + \gamma \cdot g_{ms1}\right) \cdot \frac{(1 + R_S \cdot g_{m2})^2}{R_S \cdot g_{m1}^2}$$

Final form after simplifications

$$A_V = -g_{m1} \cdot R_L = 1 - \frac{1}{g_{m2} \cdot R_S} < 0$$

$$\frac{1}{(1 - A_V)} = g_{m2} \cdot R_S$$

$$\frac{P_{OUT}}{P_{SOURCE}} = \frac{V_{OUT}^2}{2 \cdot Z_{OUT}} \cdot \frac{2 \cdot R_S}{\underbrace{V_{SOURCE}^2}_{4 \cdot V_{IN}^2}} = \frac{A_V^2 \cdot R_S}{4} \cdot \frac{2}{R_L} \cdot \frac{1 - A_V}{2 - A_V} \approx \frac{g_{m2}^2 \cdot R_L \cdot R_S}{2}$$

$$F = 1 + \gamma \cdot g_{ms2} \cdot R_S + (1 + \gamma \cdot g_{ms1} \cdot R_L) \cdot \frac{(1 + R_S \cdot g_{m2})^2}{R_L \cdot R_S \cdot g_{m1}^2}$$

$$F = 1 + \gamma \cdot \overbrace{\frac{1}{\underbrace{\boxed{(1 - A_V)}}_{>>1}}}^{\text{M2 contrib}} + \overbrace{\underbrace{(1 - \gamma \cdot A_V)}_{>>1} \cdot \frac{R_L}{R_S}}^{\text{RL and M1 contrib}} \cdot \frac{1}{A_V^2} \cdot \underbrace{\left(\frac{2 - A_V}{1 - A_V}\right)^2}_{\approx 1} \approx 1 - \frac{\gamma}{A_V} \cdot \left(1 + \frac{R_L}{R_S}\right)$$

$$Z_O = \frac{R_L \cdot (1 + R_S \cdot g_{m2})}{2} = \frac{R_L}{2} \cdot \frac{(2 - A_V)}{(1 - A_V)}$$

Design summary

- Setting the voltage gain, $gm1 \cdot RL$ imposes gm2
- Choosing gm1 and RL impacts consumption and NF
- Compute power gain and NF for various values of RL, different gain to see impact.
- The gain will be limited by the CL at the output and the voltage drop across RL, going to stronger inversion increases $RL \cdot ID1$ by $1/g(IC)$
- Find the best trade-off!
- E.g. RL=100Ω, in WI: ID1=3mA, ID2=70uA, PG=13dB, F=1.2